Mathematics Public Lesson 2 "Volume of Tetrahedron with congruent faces"

Date of the lesson: Wednesday, January 18, 2006
Teacher: Ohneda, Hiroshi

1. Class Lower Secondary Grade 3 (Grade 9), Homeroom No.4 (21Boys, 20Girls)
2. Introduction

Instead of viewing mathematics learning as a passive process of receiving mathematical knowledge as cultural products from the teacher to students, we should consider it as an active process with teacher support, where students possess the disposition, "Let's make new mathematics." Therefore, I believe it is important for teachers to continuously strive to identify factors that support the establishment of mathematical activities in a lesson that facilitate students' thinking.

It is important that students feel pleasant about the experiences of such mathematical activities. It is essential that mathematics lessons meet students' intrinsic needs. It is my hope that I can enjoy mathematics with students by developing lessons that will allow students to feel the "benefits and pleasure of mathematics,"

3. Perspective on instructional material

As a result of the reduction of the number of mathematics lessons in the current National Course of Study, the topic, cutting and projecting of solid figures, has been eliminated. However, as the phrases such as "enjoying mathematical activities," and "observing, manipulating and experimenting" suggest mathematical ways of observing and thinking reflected in cutting and projecting solid figures are absolutely necessary and must be taught regardless. Thus, although these topics have been eliminated from the Course of Study, teachers are still expected to provide the necessary instruction. I am very concerned about the status of such a foundational content for developing spatial sense.

For example, suppose you received a present in a little box. You will be very excited and wonder, "What is my present?" You will imagine various items that might be in the box. Based on the size, weight, color, sound, etc., you try to determine what might be in the box. If the present is a jewelry, you will look at it from a variety of angles and observe its shape, brightness, and design. When we try to recognize an object, we observe it from different angles and do a variety of things to analyze what it is made of and how it is shaped. In that process, the mathematical ways of observing and thinking reflected in cutting and projecting solid figures become necessary.

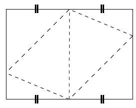
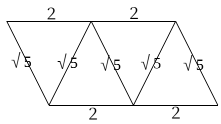
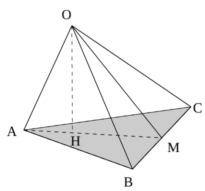
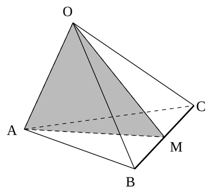
4. Goals of today's lesson and lesson plan

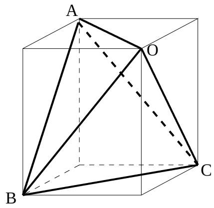
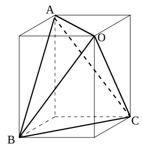
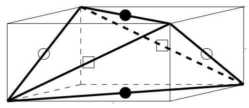
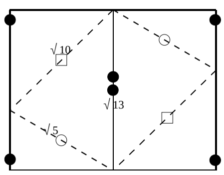
At our school, we continue to develop a geometry curriculum recognizing the important role of space in the domain of geometry. In the second grade of lower secondary school (Grade 8), we plan a series of lessons centered around the notion of projections of space figures. In the third grade (Grade 9), our lessons will explore measurements of space figures, incorporating various mathematical activities. At this point of the school year, we have already completed the required contents of the third grade (Grade 9); therefore, we plan lessons that are more topical as enrichment.

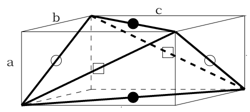
The first goal of today's lesson is to make students become interested in the exploration by folding papers to construct tetrahedron with congruent faces. Because the students will have the actual tetrahedron by their sides, they will be more easily imagine the process of measurement concretely, and mathematical activities will be facilitated. Furthermore, by thinking about solid tetrahedron with congruent faces, we attempt to shift the way students may be looking at the problem so that they can understand the way the solids are structured more deeply.

The emphasis here is to go beyond looking at geometric figures analytically and to deepen their examination of space figures further by considering the viewpoint of actually making the object.

5. Flow of the lesson

Content	Instructional Activities	Points of consideration & evaluation
Introduction of the task Actually fold the given design to confirm that the space figure is a tetrahedron with congruent faces.	Display the task on the blackboard. Examine what kind of space figure by actually folding the paper. <Task 1>> What kind of space figure will you have if you folded the design shown on the right?  By substituting numerical values, affirm that the faces are congruent isosceles triangles. → Understand the phrase, “Tetrahedron with congruent faces.”	By having students engage in paper folding, promote interest in the task and facilitate mathematical activity.
Understanding the task Determining the volume of a tetrahedron with congruent faces.	<<Task 2>> Determine the volume of the tetrahedron with congruent faces you can construct from the figure shown on the right.  Anticipated students' solutions  We know that $V = \triangle ABC \times OH \div 3$ but how can we calculate… → Any symple way? [Solution 1] △ ABC is an equilateral triangle with each side 2 unit long. Therefore, $OH = \sqrt{3}$ Consequently, $V = \triangle ABC \times OH \div 3$ $= (AM \times BC \div 2) \times \sqrt{3} \div 3$ $= 2\sqrt{3} / 3$ [Solution 2] If we cut the tetrahedron by plane OAM, we obtain two congruent triangular prisms. Therefore,  $V = (\triangle OAM \times BM \div 3) \times 2$ $= \triangle OAM \times BC \div 3$ $= \sqrt{3} \times 2 \div 3$	By having an actual tetrahedron with congruent faces, students might be able to more easily have an image of the task, facilitating mathematical activity. To support students’ note taking and as a focus point for thinking, record this on the blackboard. To avoid careless computational error, make sure that students' writing is clear and easy to understand. → An important step toward high school mathematics.

	[Solution 3] The given tetrahedron with congruent faces can be obtained by cutting a square prism as shown in the figure below. Therefore,  $V =$ Volume of Cube $\div 3$ $= ($Area of Base $\times$ Height $\div 3$ $= 2 \times \sqrt{3} \div 3$ When we made a tetrahedron with congruent faces by folding paper, inside was empty. How can we make such a tetrahedron that is solid? Anticipated student response → We can cut a wooden square prism. So, … [Solution 2]  	If this solution is not proposed, proceed to the next task. Omit [Cutting of a cube] (below) if [Solution 3] is proposed.
Enrichment Task 1 Cutting of a cube	<<Task 3>> If we cut a cube (for example an erase), what kind of tetrahedron with congruent face can we make? Anticipated students response → We can get a tetrahedron with congruent faces that are scalene triangles. 	Limit the discussion to a brief explanation. To support students ' note taking and as a focus point for thinking, record this on the blackboard.
Enrichment Task 2 Constructing a model of tetrahedron with congruent faces	By folding arbitrary rectangular papers, make the tetrahedron with congruent faces shown above. <<Task 4>> Verify that you can make the given tetrahedron with congruent faces by folding an arbitrary rectangular paper.	Understand that the tetrahedron with congruent faces can be made by folding an paper.
Enrichment task 3 Calculating the volume of various tetrahedral with congruent faces	Provide different set of measurements and calculate the volume. <<Task 5>> Calculate the volume of a tetrahedron with the edges with lengths $\sqrt{5}$, $\sqrt{10}$, and $\sqrt{13}$. Anticipated students response: Let the lengths of the edges of the rectangular prism be *a*, *b*, and *c*. Then,  	Advanced students might simply calculate the volume, but make sure that their writing is clear and easy to understand.

	$a^2+b^2=5$ $b^2+c^2=13$ $c^2+a^2=10$  By solving this system of equations, $a=1,\ b=2,\ c=3$ Therefore, $V=(1\times 2\times 3)\div 3$ $=2$		